**Statement:** Use linkage transformation on the linkage of Figure P2-1a to make it a 1-DOF mechanism.

**Solution:** See Figure P2-1a and Mathcad file P0209.

1. The mechanism in Figure P2-1a has mobility:

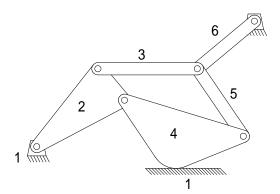
Number of links 
$$L := 6$$

Number of full joints 
$$J_1 = 7$$

Number of half joints 
$$J_2 = 1$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 0$$



2. Use rule 2, which states: "Any full joint can be replaced by a half joint, but this will increase the *DOF* by one." One way to do this is to replace one of the pin joints with a pin-in-slot joint such as that shown in Figure 2-3c. Choosing the joint between links 2 and 4, we now have mobility:

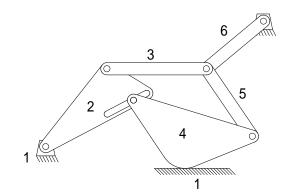
Number of links 
$$L := 6$$

Number of full joints 
$$J_1 = 6$$

Number of half joints 
$$J_2 = 2$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



**Statement:** Find the mobility of the mechanisms in Figure P2-4.

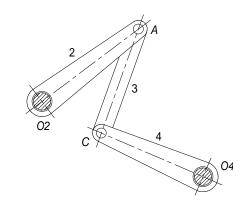
**Solution:** See Figure P2-4 and Mathcad file P0221.

- 1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility.
- a. This is a basic fourbar linkage. The input is link 2 and the output is link 4. The cross-hatched pivot pins at *O*2 and *O*4 are attached to the ground link (1).

Number of links 
$$L = 4$$
  
Number of full joints  $J_1 = 4$   
Number of half joints  $J_2 = 0$ 

$$M\coloneqq 3{\cdot}(L-1)-2{\cdot}J_1-J_2$$

$$M = 1$$



b. This is a fourbar linkage. The input is link 2, which in this case is the wheel 2 with a pin at A, and the output is link 4. The cross-hatched pivot pins at O2 and O4 are attached to the ground link (1).

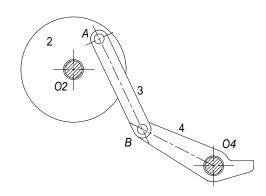
Number of links 
$$L := 4$$

Number of full joints 
$$J_1 = 4$$

Number of half joints 
$$J_2 = 0$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



c. This is a 3-cylinder, rotary, internal combustion engine. The pistons (sliders) 6, 7, and 8 drive the output crank (2) through piston rods (couplers 3, 4, and 5). There are 3 full joints at the crank where rods 3, 4 and 5 are pinned to crank 2. The cross-hatched crank-shaft at *O*2 is supported by the ground link (1) through bearings.

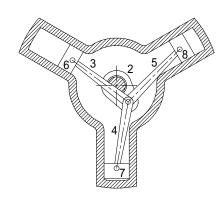
Number of links 
$$L = 8$$

Number of full joints 
$$J_1 = 10$$

Number of half joints 
$$J_2 = 0$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

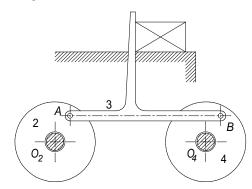
$$M = 1$$



d. This is a fourbar linkage. The input is link 2, which in this case is a wheel with a pin at A, and the output is the vertical member on the coupler, link 3. Since the lengths of links 2 and 4 ( $O_2A$  and  $O_4B$ ) are the same, the coupler link (3) has curvilinear motion and AB remains parallel to  $O_2O_4$  throughout the cycle. The cross-hatched pivot pins at  $O_2$  and  $O_4$  are attached to the ground link (1).

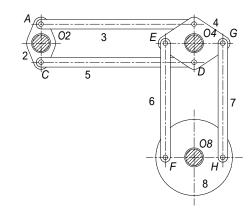
Number of links 
$$L := 4$$
  
Number of full joints  $J_1 := 4$   
Number of half joints  $J_2 := 0$   
 $M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$ 

M = 1



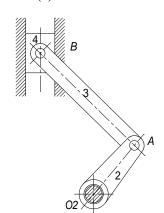
e. This is a fourbar linkage with an output dyad. The input (rocker) is link 2 and the output (rocker) is link 8. Links 5 and 6 are redundant, i.e. the mechanism will have the same motion if they are removed. The input fourbar consists of links 1, 2, 3, and 4. The output dyad consists of links 7 and 8. The cross-hatched pivot pins at *O*2, *O*4 and *O*8 are attached to the ground link (1). In the calculation below, the redundant links and their joints are not counted (subtract 2 links and 4 joints from the totals).

Number of links 
$$L := 6$$
  
Number of full joints  $J_1 := 7$   
Number of half joints  $J_2 := 0$   
 $M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$   
 $M = 1$ 



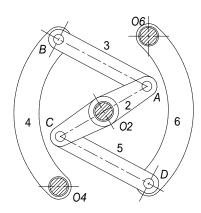
f. This is a fourbar offset slider-crank linkage. The input is link 2 (crank) and the output is link 4 (slider block). The cross-hatched pivot pin at *O*2 is attached to the ground link (1).

Number of links 
$$L := 4$$
  
Number of full joints  $J_1 := 4$   
Number of half joints  $J_2 := 0$   
 $M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$   
 $M = 1$ 



g. This is a fourbar linkage with an alternate output dyad. The input (rocker) is link 2 and the outputs (rockers) are links 4 and 6. The input fourbar consists of links 1, 2, 3, and 4. The alternate output dyad consists of links 5 and 6. The cross-hatched pivot pins at *O*2, *O*4 and *O*6 are attached to the ground link (1).

Number of links 
$$L := 6$$
  
Number of full joints  $J_1 := 7$   
Number of half joints  $J_2 := 0$   
 $M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$   
 $M = 1$ 

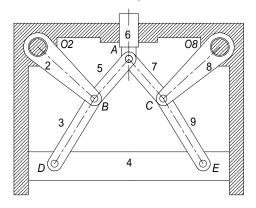


h. This is a ninebar mechanism with three redundant links, which reduces it to a sixbar. Since this mechanism is symmetrical about a vertical centerline, we can split it into two mirrored mechanisms to analyze it. Either links 2, 3 and 5 or links 7, 8 and 9 are redundant. To analyze it, consider 7, 8 and 9 as the redundant links. Analyzing the ninebar, there are two full joints at the pins *A*, *B* and *C* for a total of 12 joints.

Number of links 
$$L = 9$$
  
Number of full joints  $J_1 = 12$   
Number of half joints  $J_2 = 0$ 

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 0$$

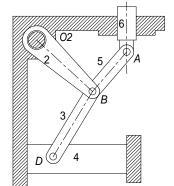


The result is that this mechanism seems to be a structure. By splitting it into mirror halves about the vertical centerline the mobility is found to be 1. Subtract the 3 redundant links and their 5 (6 minus the joint at A) associated joints to determine the mobility of the mechanism.

Number of links 
$$L := 9 - 3$$
  
Number of full joints  $J_1 := 12 - 5$   
Number of half joints  $J_2 := 0$ 

$$M \coloneqq 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



**Statement:** Find the mobility of the mechanisms in Figure P2-5.

**Solution:** See Figure P2-5 and Mathcad file P0224.

1. Use equation 2.1c (Kutzbach's modification) to calculate the mobility. In the kinematic representations of the linkages below, binary links are depicted as single lines with nodes at their end points whereas higher order links are depicted as 2-D bars.

a. This is a sixbar linkage with 4 binary (1, 2, 5, and 6) and 2 ternary (3 and 4) links. The inverted U-shaped link at the top of Figure P2-5a is represented here as the binary link 6.

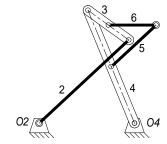
Number of links 
$$L := 6$$

Number of full joints 
$$J_1 = 7$$

Number of half joints 
$$J_2 = 0$$

$$M \coloneqq 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



b. This is an eightbar linkage with 4 binary (1, 4, 7, and 8) and 4 ternary (2, 3, 5, and 6) links. The inverted U-shaped link at the top of Figure P2-5b is represented here as the binary link 8.

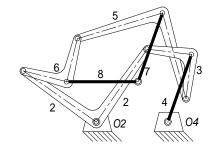
Number of links 
$$L := 8$$

Number of full joints 
$$J_1 = 10$$

Number of half joints 
$$J_2 = 0$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



**Statement:** Find the mobility of the automotive throttle mechanism shown in Figure P2-7.

**Solution:** See Figure P2-7 and Mathcad file P0226.

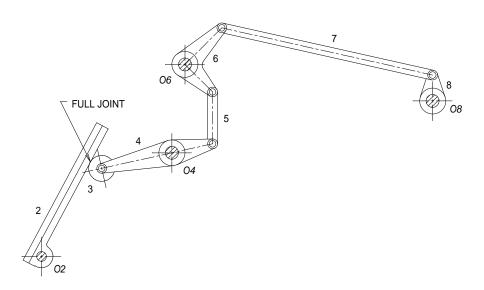
1. This is an eightbar linkage with 8 binary links. It is assumed that the joint between the gas pedal (2) and the roller (3) that pivots on link 4 is a full joint, i.e. the roller rolls without slipping. The pivot pins at *O*2, *O*4, *O*6, and *O*8 are attached to the ground link (1). Use equation 2.1c (Kutzbach's modification) to calculate the mobility.

Number of links L = 8

Number of full joints  $J_1 = 10$ 

Number of half joints  $J_2 = 0$ 

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2 \qquad M = 1$$



**Statement:** Find the mobility of the corkscrew in Figure P2-9.

**Solution:** See Figure P2-9 and Mathcad file P0228.

1. The corkscrew is made from 4 pieces: the body (1), the screw (2), and two arms with teeth (3), one of which is redundant. The second arm is present to balance the forces on the assembly but is not necessary from a kinematic standpoint. So, kinematically, there are 3 links (body, screw, and arm), 2 full joints (sliding joint between the screw and the body, and pin joint where the arm rotates on the body), and 1 half joint where the arm teeth engage the screw "teeth". Using equation 2.1c, the *DOF* (mobility) is

Number of links L := 3

Number of full joints  $J_1 = 2$ 

Number of half joints  $J_2 = 1$ 

 $M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2 \qquad M = 1$ 

**Statement:** Figure P2-11 shows a bicycle hand brake lever assembly. Sketch a kinematic diagram of this

device and draw its equivalent linkage. Determine its mobility. Hint: Consider the flexible cable

to be a link.

**Solution:** See Figure P2-11 and Mathcad file P0230.

1. The motion of the flexible cable is along a straight line as it leaves the guide provided by the handle bar so it can be modeled as a translating full slider that is supported by the handlebar (link 1). The brake lever is a binary link that pivots on the ground link. Its other node is attached through a full pin joint to a third link, which drives the slider (link 4).

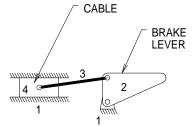
Number of links 
$$L = 4$$

Number of full joints 
$$J_1 = 4$$

Number of half joints 
$$J_2 = 0$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 1$$



Statement:

Figure P2-12 shows a bicycle brake caliper assembly. Sketch a kinematic diagram of this device and draw its equivalent linkage. Determine its mobility under two conditions.

- Brake pads not contacting the wheel rim.
- Brake pads contacting the wheel rim.

Hint: Consider the flexible cable to be replaced by forces in this case.

Solution: See Figure P2-12 and Mathcad file P0231.

1. The rigging of the cable requires that there be two brake arms. However, kinematically they operate independently and can be analyzed that way. Therefore, we only need to look at one brake arm. When the brake pads are not contacting the wheel rim there is a single lever (link 2) that is pivoted on a full pin joint that is attached to the ground link (1). Thus, there are two links (frame and brake arm) and one full pin joint.

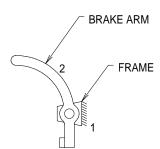
Number of links 
$$L = 2$$

Number of full joints 
$$J_I = 1$$
  
Number of half joints  $J_2 = 0$ 

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

 $J_2 = 0$ 

$$M = 1$$



When the brake pad contacts the wheel rim we could consider the joint between the pad, which is rigidly attached to the brake arm and is, therefore, a part of link 2, to be a half joint. The brake arm (with pad), wheel (which is constrained from moving laterally by the frame), and the frame constitute a structure.

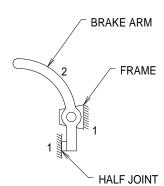
Number of links 
$$L := 2$$

Number of full joints 
$$J_1 = 1$$

Number of half joints 
$$J_2 = 1$$

$$M := 3 \cdot (L-1) - 2 \cdot J_1 - J_2$$

$$M = 0$$

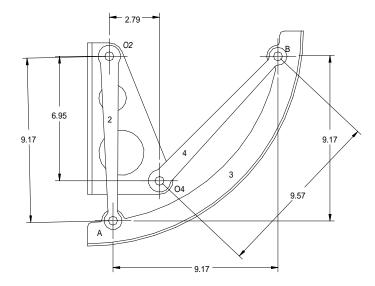


**Statement:** Find the mobility, Grashof condition and Barker classification of the aircraft overhead bin shown in Figure P2-19.

**Solution:** See Figure P2-19 and Mathcad file P0243.

1. Use inequality 2.8 to determine the Grashof condition and Table 2-4 to determine the Barker classification.

$$Condition(a,b,c,d) \coloneqq \begin{cases} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \text{ "Grashof"} \quad if \quad SL < PQ \\ return \text{ "Special Grashof"} \quad if \quad SL = PQ \\ return \text{ "non-Grashof"} \quad otherwise \end{cases}$$



This is a basic fourbar linkage. The input is the link 2 and the output is link 4. The mobility (*DOF*) is found using equation 2.1c (Kutzbach's modification):

Number of links L := 4

Number of full joints  $J_1 = 4$ 

Number of half joints  $J_2 = 0$   $M = 3 \cdot (L-1) - 2 \cdot J_1 - J_2$  M = 1

The link lengths and Grashof condition are

$$L_I := \sqrt{2.79^2 + 6.95^2}$$
  $L_I = 7.489$   $L_2 := 9.17$ 

$$L_3 := \sqrt{9.17^2 + 9.17^2}$$
  $L_3 = 12.968$   $L_4 := 9.57$ 

 $Condition(L_1, L_2, L_3, L_4) =$ "non-Grashof"

This is a Barker Type 7 RRR3 (non-Grashof, longest link is coupler).